Std: 11 Chapter – **2 Relations and Functions** Sub: MATHS

Exercise 2.1

1. **If** $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$, **find the values of** x **and** y.

Ans. Here $\left(\frac{x}{3}+1, y-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$

$\Rightarrow \frac{x}{3}+1=\frac{5}{3}$ and $y-\frac{2}{3}=\frac{1}{3}$

$\Rightarrow \frac{x}{3}=\frac{5}{3}-1$ and $y=\frac{1}{3}+\frac{2}{3}$

$\Rightarrow \frac{x}{3}=\frac{2}{3}$ and $y=\frac{3}{3}$

$\Rightarrow x=2$ and $y=1$

2. **If the set A has 3 elements and the set B = {3, 4, 5}, then find the number of elements in (A × B).**

Ans. Number of elements in set A = 3 and Number of elements in set B = 3

$\therefore$ Number of elements in A × B = 3 × 3 = 9

3. **If G = {7, 8} and H = {5, 4, 2}, find G × H and H × G.**

Ans. Given: G = {7, 8} and H = {5, 4, 2}

$\therefore$ G × H = {(7,5), (7,4), (7,2), (8,5), (8,4), (8,2)}

And H × G = {(5,7), (4,7), (2,7), (5,8), (4,8), (2,8)}

4. **State whether each of the following statements are true or false. If the statement is false,**

rewrite the given statement correctly:

(i) If P = $\{m, n\}$ **and Q =** $\{n, m\}$, **then P** $\times$ **Q =** $\{(m, n)(n, m)\}$.

(ii) If A and B are non-empty sets, then A $\times$ **B is a non-empty set of ordered pairs** (x, y) **such that** $x \in$ **A and** $y \in$ **B.**

(iii) **If A = {1, 2}, B = {3, 4}, then** $\mathrm{A} \times (\mathrm{B} \cap \phi) = \phi$

Ans. (i) Here P = $\{m, n\}$ and Q = $\{n, m\}$

Number of elements in set P = 2 and Number of elements in set Q = 2

$\therefore$ Number of elements in P $\times$ Q = 2 $\times$ 2 = 4

But P $\times$ Q = $\{(m, n), (n, m)\}$ and here number of elements in P $\times$ Q = 2 Therefore, statement is false.

Correct statment is $P \times Q = \{(m, m), (n, n), (n, m), (m, n)\}$

(ii) True

(iii) True

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5. **If A =** $\{-1, 1\}$, **find A** $\times$ **A** $\times$ **A.**

Ans. Here A = $\{-1, 1\}$

A $\times$ A = $\{(-1, -1), (-1, 1), (1, -1), (1, 1)\}$

$\therefore$ A $\times$ A $\times$ A = $\{(-1, -1, -1), (-1, -1, 1), (-1, 1, -1), (-1, 1, 1), (1, -1, -1), (1, -1, 1), (1, 1, -1), (1, 1, 1)\}$

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6. **If A** $\times$ **B =** $\{(a, x), (a, y), (b, x), (b, y)\}$, **find A and B.**

Ans. Given: A $\times$ B = $\{(a, x), (a, y), (b, x), (b, y)\}$

$\therefore$ A = set of first elements = $\{a, b\}$ and B = set of second elements = $\{x, y\}$

7. Let A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}. Verify that:

(i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(ii) A $\times$ C is a subset of B $\times$ D.

Ans. Given: A = {1, 2}, B = {1, 2, 3, 4}, C =

{5, 6} and D = {5, 6, 7, 8}

(i) $B \cap C$ = {1, 2,3, 4} $\cap$ {5, 6}= ϕ

$\therefore A \times B \cap C = \{1, 2\} \times \phi = \phi$ ....(i)

A $\times$ B = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)}

A $\times$ C = {(1,5), (1,6), (2,5), (2,6) (A

$\therefore$ $\times$ B) $\cap$ (A $\times$ C)= ϕ ....(ii)

Therefore, from eq. (i) and (ii), $A \times B \cap C$

= (A $\times$ B) $\cap$ (A $\times$ C)

(ii) A $\times$ C = {(1, 5), (1, 6), (2, 5), (2, 6)

B $\times$ D = {(1,5), (1,6), (1,7), (1,8), (2,5), (2,6), (2,7), (2,8), (3,5), (3,6), (3,7), (3,8),

(4, 5), (4, 6), (4, 7), (4, 8),

Therefore, it is clear that each element of A $\times$ C is present in B $\times$ D. A $\times$ C

$\therefore$ $\subset$ B $\times$ D

8. Let A = {1, 2} and B = {3, 4}, write A $\times$ B. How many subsets will A $\times$ B have? List them.

Ans. Given: A = {1, 2} and B = {3, 4}

$\therefore$ $A \times B = \{(1,3), (1,4), (2,3), (2,4)\}$

Number of elements in $A \times B = 4$

Therefore, Number of subsets of $A \times B = 2^4 = 16$

ϕ ,{(2,3)},{(1,4)},{(2,3)},{(2,4)},{(1,3),(1,4)},{(1,3),(2,3)},{(1,3),(2,4)},{(1,4),(2,3)},{(1,4),(2,4)},{(2,3),(2,4)},{(1,3),(1,4),(2,3)},{(1,3),(1,4),(2,4)},{(1,3),(2,3),(2,4)},{(1,4),(2,3),(2,4)},{(1, 3), (1, 4), (2, 3), (2, 4)}

9. Let A and B be two sets such that $n(A) = 3$ and $n(B) = 2$. If $(x,1),(y,2)(z,1)$ are in $A \times B$.

Ans. Here $(x,1) \in A \times B$

$\Rightarrow x \in A$ and $1 \in B$

$(y,2) \in A \times B$

$\Rightarrow y \in A$ and $2 \in B$

$(z,1) \in A \times B$

$\Rightarrow z \in A$ and $1 \in B$

But it is given that $n(A) = 3$ and $n(B) = 2$

$\therefore$ $A = \{x, y, z\}$ and B = {1, 2}

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10. The Cartesian Product $A \times A$ has 9 elements among which are found $(-1, 0)$ and (0, 1). Find the set A and the remaining elements of $A \times A$.

Ans. Here $(-1, 0) \in A \times A$

$\Rightarrow -1 \in A$ and $0 \in A$

$(0,1) \in A \times A$

$\Rightarrow 0 \in$ A and $1 \in$ A

$\therefore -1, 0, 1 \in$ A

But it isgiven that $n(A \times A) = 9$ which implies that $n(A) = 3$

$\therefore$ A= $\{-1, 0, 1\}$

And A$\times$ A= $\{(-1,-1),(-1,0),(-1,1),(0,-1),(0,0),(0,1),(1,-1),(1,0),(1,1)\}$

Therefore, the remaining elements of A $\times$ A are

(-1,-1),(-1,1),(0,-1),(0,0),(1,-1),(1,0),(1,1)

Exercise 2.2

1. Let A = {l, 2, 3, ..., 14}. Define a relation R from A to A by R = $\{(x, y): 3x-y=0,$ where $x, y \in A\}$. Write down its domain co-domain and range.

Ans. Given: A = {1, 2, 3, ..., 14}

The ordered pairs which satisfy $3x-y=0$ are (1, 3), (2, 6), (3, 9) and (4, 12).

$\therefore$ R = {(1, 3), (2, 6), (3, 9), (4, 12)}

Domain = {1, 2, 3, 4}

Range = {3, 6, 9, 12}

Co-domain = {1, 2, 3, ..., 14}

2. Define a relation R on the set N of natural numbers R = $\{(x, y): y=x+5, x$ is a natural number less than 4: $x, y \in N\}$. Depict this relationship using roster form. Write down the domain and the range.

Ans. Given: R = $\{(x, y: y=x+5, x \text{ is a natural number less than } 4: x, y \in N)\}$ Putting x =1, 2, 3 in $y=x+5$, we get y = 6, 7, 8

$\therefore$ R = {(1, 6), (2, 7), (3, 8)}

Domain = {1, 2, 3}

Range = {6, 7, 8}

3. A = {l, 2, 3 5} and B = {4, 6, 9}. Define a relation R from A to B by R = $\{(x, y)$: the difference between x and y is odd: $x \in A, y \in B\}$. Write R in roster form. **Ans.** Given: A = {1, 2, 3, 5} and B = {4, 6, 9}, $x \in$ A, $y \in$ B

$\therefore x-y$ = (1 – 4), (1 – 6), (1 – 9), (2 – 4), (2 – 6), (2 – 9), (3 – 4), (3 – 6) (3 – 9), (5 – 4), (5 – 6), (5 – 9)

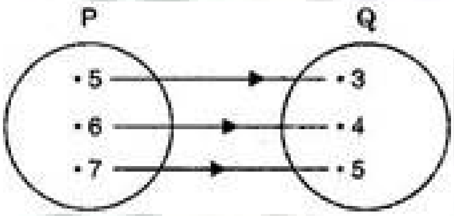
$\Rightarrow x-y=-3,-5,-8,-2,-4,-7,-1,-3,-6,1,-1,-4$

$\therefore$ R = {(1, 4), (1, 6), (2, 9), (3, 4), (3, 6) (5, 4), (5, 6)}

4. Figure shows a relationship between the sets P and Q. Write this relation:

(i) in set-builder form

(ii) roster form



What is its domain and range?

Ans. (i) Relation R in set-builder form is R = $\{(x,y): y=x-2: x=5,6,7\}$

(ii) Relation R in roster form is R = {(5, 3), (6, 4), (7, 5)} Domain =

{5, 6, 7}

Range = {3, 4, 5}

5. Let A = {1, 2, 3, 4, 6}. Let R be the relation on A defined by $\{(a,b): a,b \in A, b$ is exactly divisible by $a\}$.

(i) Write R in roster form.
(ii) Find the domain of R.

(iii) Find the range of R.

Ans. Given: A = {1, 2, 3, 4,

6}

A set of ordered pairs (a, b) where b is exactly divisible by a.

(i) R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)}

(ii) Domain of R = {1, 2, 3, 4, 6}

(iii) Range of R = {1, 2, 3, 4, 6}

6. Determine the domain and range of the relation R defined by R= $\{(x, x+5) : x \in (0,1,2,3,4,5)\}$

Ans. Given: R = $\{(x, x+5) : x \in (0,1,2,3,4,5)\}$ = $\{(a, b) : a = 0,1,2,3,4,5\}$

$\therefore a = x$ and $b = x+5$

Putting $a = 0, 1, 2, 3, 4, 5$ weget $b = 5, 6, 7, 8, 9, 10$

$\therefore$ Domain of R = {0,1,2,3,4 5}

Range of R = {5,6,7,8,9,10}

7. Write the relation R = $\{(x, x^3) : x$ **is a prime number less than** $10\}$ **in roster form.**

Ans. Given: R = $\{(x, x^3) : x \text{ is a prime number less than } 10\}$

Putting x = 2, 3, 5, 7

R = {(2, 8), (3, 27),(5, 125), (7, 343)}

8. Let A= $\{x, y, z\}$ **and B = {1, 2}. Find the number of relations from A to B.**

Ans. Given: A = $\{x, y, z\}$ and B = {1, 2}

Number of elements in set A = 3 and Number of elements in set B = 2 Number of

$\therefore$ subsets of $A \times B = 3 \times 2 = 6$

Number of relations from A to $B = 2^6$.

9. Let R be the relation on Z defined by R = {(a,b):a,b$\in Z$ is an integer}. Find the domain and range of R.

Ans. Given: R= $\{(a,b): a,b \in Z,\ a-b \text{ is an integer}\}$

= $\{(a,b): a,b \in Z, \text{ both } a \text{ and } b \text{ are even or both } a \text{ and } b \text{ are odd}\}$

= $\{(a,b): a,b \in Z,\ (a \text{ and } b \text{ are even}) \cup (a \text{ and } b \text{ are odd})\}$

$\therefore$ Domain of R = Z

Range of R = Z

Exercise 2.3

1. Which of the following are functions? Give reasons. If it is a function determine its domain and range.

(i) {(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}

(ii) {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}

(iii) {(1, 3), (1, 5), (2, 5)}

Ans. (i) Given Relation is {(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)}

All values of x are distinct. Each value of x has a unique value of y.

Therefore, the relation is a function.

$\therefore$ Domain of function = {2, 5, 8, 11, 14, 17}

Range of function = {1}

(ii) Given: Relation is {(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}

All values of x are distinct. Each value of x has a unique value of y.

Therefore, the relation is a function.

$\therefore$ Domain of function = {2, 4, 6, 8, 10, 12, 14}

Range of function = {1, 2, 3, 4, 5, 6, 7}

(iii) Given: Relation is {(1, 3), (1, 5), (2, 5)}

This relation is not a function because there is an element 1 which is associated to two elements 3 and 5.

2. Find the domain and range of the following real functions:

(i)
(ii) $f(x)=\sqrt{9-x^2}$

Ans. (i) given $f(x)=-|x|$ The function is defined for all real values of x.

$\therefore$ Domain of the function = R

Now, when $x<0$, then $|x|=-x$

$\therefore f(x)=-(-x), x<0$

When

$\therefore f(x)=-|0|=0$

$x=0, |x|=0$

When

x.

$x>0, |x|=x$

$\therefore f(x)=-x<0$

Therefore, $f(x)\le 0$ for all real values

$\therefore$ of Range of function = $(-\infty, 0]$

(ii) Given: $f(x)=\sqrt{9-x^2}$.

The function is not defined when $9-x^2<0$.

$\therefore$ Domain of function = $\{x: 9-x^2\ge 0\}=\{x: x^2-9\le 0\}$

$f(x)=-|x|$

= $\{x : (x+3)(x-3) \le 0\}$ = [-3,3]

$\therefore$ Range of function = [0,3]

3. A function f is defined by $f(x) = 2x - 5$. Write down the values of

(i) $f(0)$ (ii) $f(7)$ (iii) $f(-3)$

Ans. Given: $f(x) = 2x - 5$

(i) Putting $x = 0$,

$f(0) = 2 \times 0 - 5 = -5$

(ii) Putting $x = 7$,

$f(7) = 2 \times 7 - 5 = 14 - 5 = 9$

(iii) Putting $x = -3$,

$f(-3) = 2 \times (-3) - 5 = 6 - 5 = -11$

4. The function t which maps temperature in degree Celsius into temperature in degree Fahrenheit is defined by $t(C) = \frac{9C}{5} + 32$. Find:

(i) $t(0)$

$t(28)$

(ii) $t(-10)$

(iv) The value of C when

$t(C) = 212$. **Ans.** Given:

$$t(C) = \frac{9C}{5} + 32$$

(i) Putting C = 0,

$$t(0) = \frac{9 \times 0}{5} + 32 = 32$$

(ii) Putting C = 28,

$$t(28) = \frac{9 \times 28}{5} + 32 = \frac{252 + 160}{5} = \frac{412}{5}$$

$$t(-10) = \frac{9 \times (-10)}{5} + 32 = -18 + 32 = 14$$

(iii) Putting C = –10,

(iv) Putting $t(C)$ = 212, $212 = \frac{9C}{5} + 32$

$\Rightarrow \frac{9C}{5} = 212 - 32$

$\Rightarrow \frac{9C}{5} = 180$

$\Rightarrow C = 180 \times \frac{5}{9} = 100$

5. Find the range of each of the following functions:

(i) $f(x) = 2 - 3x, x \in R, x > 0$

(ii) $f(x) = x^2 + 2, x$ **is a real number.**

(iii) $f(x) = x, x$ **is a real number.**

Ans. (i) Given: $f(x) = 2 - 3x$, $x \in$ R and $x > 0$

$\therefore 3x > 0 \Rightarrow -3x < 0$

$\Rightarrow 2 - 3x < 2$

$\therefore$ Range offunction

= $\{a \in R : a < 2\}$ = $(-\infty, 2)$

(ii) Given: $f(x) = x^2 + 2,\ x \in$ R

$\therefore x^2 \geq 0$ for $x \in$ R

$\Rightarrow x^2 + 2 \geq 2$

$\therefore$ Range of function

= $\{a \in R : a \geq 2 \forall a \in R\}$ = $[2, \infty)$

(iii) Given: $f(x) = x$, $x \in$ R

$\therefore$ Range of function = R